FACTORIZATION OF DAMPED WAVE EQUATIONS WITH CUBIC NONLINEARITY

H. C. ROSU*, O. CORNEJO-PÉREZ

Potosinian Institute of Science and Technology,

Apdo. Postal 3-74 Tangamanga, 78231 San Luis Potosí, Mexico

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Abstract. The recent factorization scheme that we introduced for nonlinear polynomial ODEs in math-ph/0401040 is applied to the interesting case of damped wave equations with cubic nonlinearities. Traveling kink solutions are possible in the plane defined by the kink velocity versus the damping coefficient only along hyperbolas that are plotted herein.

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In the cubic nonlinear wave equation with damping [1,2]

$$\frac{d^2 f}{dx_0^2} - \frac{d^2 f}{dx_1^2} + \lambda_0 \frac{df}{dx_0} - f + f^3 = 0 \tag{1}$$

we set $x_0 \to t$, $x_1 \to x$ and $\tau = x - \alpha t$ to get the following equation

$$(1-\alpha^2)\frac{d^2 f}{d\tau^2} + \alpha\lambda_0 \frac{df}{d\tau} + f - f^3 = 0 \ . \tag{2}$$

Rewriting as

$$f'' + \beta f' + g(f) = 0 \ , \tag{3}$$

where

$$\beta = \frac{\alpha\lambda_0}{1-\alpha^2}, \qquad \text{and} \qquad g(f) = f(1-f^2) \ , \tag{4}$$

we obtain the standard form of the equation allowing the application of our recent factorization scheme [3]. In general, the factorization methods lead to traveling kink solutions, which are important in many applications. When first derivatives are present in a second-order differential equation with polynomial nonlinearity, the kink solutions are obtained only for special values of the coefficient in front of the

*E-mail: hcr@ipicyt.edu.mx

first derivative. For other values, more complicated solutions occur. For example, in the case of Eq. (1), the general solutions are elliptic functions [1,2].

To apply the scheme in math-ph/0401040, we define f_1 and ϕ_2 therein as the following functions

$$f_1 = \mp\frac{1}{\sqrt{2}}(1+f), \quad \phi_2 = \mp\sqrt{2}(1-f)~. \tag{5}$$

Then, the factorization leading to the hyperbolic tangent kinks is possible for $\beta_{\rm factor} = \pm\frac{3}{\sqrt{2}}$. The two different values obtained for β yield the following equation

$$\alpha^2 \pm \frac{\sqrt{2}}{3}\lambda_0\alpha - 1 = 0 \tag{6}$$

and solutions of Eq. (6) as a function of λ_0 are given by

$$\alpha_{1,2} = \frac{1}{2}\left(-\frac{\sqrt{2}}{3}\lambda_0 \pm \sqrt{\frac{2}{9}\lambda_0^2 + 4}\right) \tag{7}$$

that is plotted in Fig. (1), and

$$\alpha_{3,4} = \frac{1}{2}\left(\frac{\sqrt{2}}{3}\lambda_0 \pm \sqrt{\frac{2}{9}\lambda_0^2 + 4}\right) \tag{8}$$

plotted in Fig. (2).

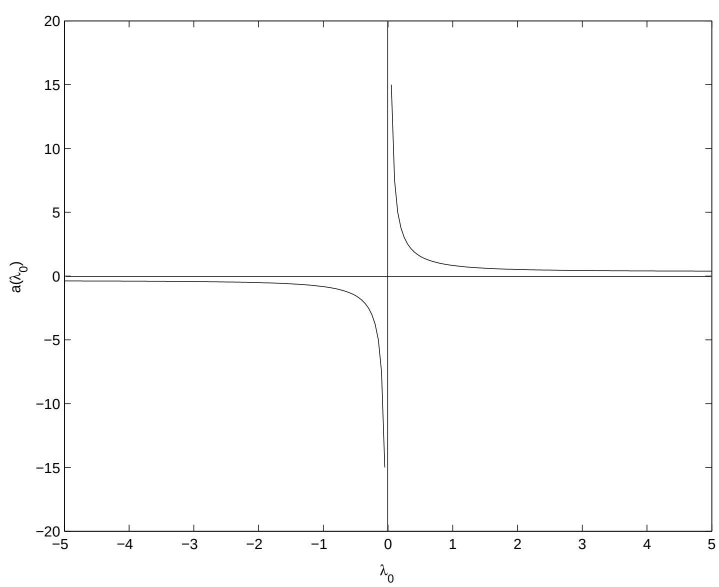

Fig. 1 The curves $\alpha_{1,2}$ in the plane (α,λ_0) along which the factorization is possible.

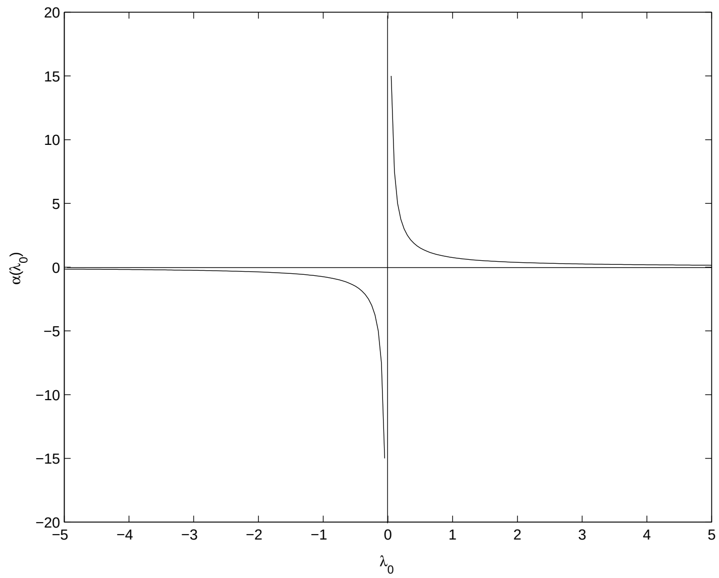

Fig. 2 The curves $\alpha_{3,4}$ in the plane (α,λ_0) along which the factorization is possible.

References

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